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SHORTER COMMUNICATION

ANALOG SOLUTION OF FREE CONVECTION MASS TRANSFER FROM DOWNWARD-FACING HORIZONTAL PLATES

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NOMENCLATURE

$\bar{F}'$, nondimensional velocity variable;
Gr_m, Grashof number;
Sc, Schmidt number;
Sh, Sherwood number;
y_{A_0}, mole fraction, diffusing component, at plate surface.

Greek symbols
$\bar{\phi}$, nondimensional concentration variable;
$\bar{\eta}$, nondimensional similarity independent variable.

1. INTRODUCTION

IN THE course of a recent study by this author [1], which demonstrated the viability of using analog computers in certain boundary layer and convection work, it was discovered that results in the paper of Bandrowski and Rybski [2] are in error. The purpose of this short communication is to give the correct results, obtained by high-speed analog computation, for the second orientation in [2], i.e. a downward-facing horizontal plate. It also serves to draw further attention to the fact that an analog computer is ideally suited to effective solution of problems of this type, a matter which appears to have been largely overlooked.

2. CORRECTED RESULTS

The problem referred to reduces to the solution of the equations

$$5Sc\,\bar{F}'''' + 3\bar{F}'''\bar{F} + \bar{F}''\bar{F}' = -2Sc\,\bar{\phi}'\bar{\eta}$$

$$5\bar{\phi}'' + 3\bar{F}\bar{\phi}' = 0$$

—equations (51) and (52) of [2]— subject to the conditions at $\bar{\eta} = 0$, $\bar{\phi}(0) = 1$; $\bar{F}'(0) = 0$;

$$\bar{F}(0) = \frac{5}{3}\frac{y_{A_0}}{1 - y_{A_0}}\,\bar{\phi}'(0)$$

and for $\bar{\eta} \to \infty$, $\bar{\phi}(\bar{\eta}) \to 0$; $\bar{F}'(\bar{\eta}) \to 0$; $\bar{F}''(\bar{\eta}) \to 0$.

In [2] the solution was accomplished digitally, and the results presented in Table 3 and Figs. 13–15. Table 3 displayed the values found for the "missing" initial conditions $\bar{F}'''(0)$, $\bar{F}''(0)$ and $\bar{\phi}'(0)$, for nine combinations of Sc and y_{A_0}; it also showed spot values for the velocity $\bar{F}'$ and concentration $\bar{\phi}$ at $\bar{\eta} = 1$. All values were quoted to five figures of precision.

In this note, values are reported for these quantities which differ considerably from the values in [2]. In the interest of brevity, the comparison is limited to the values for only three combinations of the parameters Sc and y_{A_0}; see Table 1.

As is evident, the results initially reported are substantially in error. For example, the average magnitude of the error in the velocity $\bar{F}'(1)$ is 49.8% and the average magnitude of the error in the concentration $\bar{\phi}(1)$ is 12.5%. The results surely do not warrant five figures of precision when they are subject to such errors.

The principal reason for the errors is clear from Figs. 13 and 14 of [2]. The authors failed to run the solutions out to a sufficiently large value of $\bar{\eta}$. They presumed that $\bar{\eta} = 4$ was adequate to meet the conditions as $\bar{\eta} \to \infty$, but the analog

Table 1. Comparison of results

For $Sc = 2.5$ and $y_{A_0} = 0.0005$

	$\bar{F}'''(0)$	$\bar{F}''(0)$	$\bar{\phi}'(0)$	$\bar{F}'(1)$	$\bar{\phi}(1)$
From [2]	−0.41244	0.50482	−0.33589	0.30402	0.66758
Analog results	−0.586	0.887	−0.410	0.598	0.593

For $Sc = 2.5$ and $y_{A_0} = 0.1$

	$\bar{F}'''(0)$	$\bar{F}''(0)$	$\bar{\phi}'(0)$	$\bar{F}'(1)$	$\bar{\phi}(1)$
From [2]	−0.40811	0.50885	−0.31797	0.30903	0.67979
Analog results	−0.579	0.889	−0.383	0.604	0.615

For $Sc = 100$ and $y_{A_0} = 0.01$

	$\bar{F}'''(0)$	$\bar{F}''(0)$	$\bar{\phi}'(0)$	$\bar{F}'(1)$	$\bar{\phi}(1)$
From [2]	−0.39484	0.48975	−0.33272	0.29785	0.67010
Analog results	−0.533	0.879	−0.418	0.614	0.586

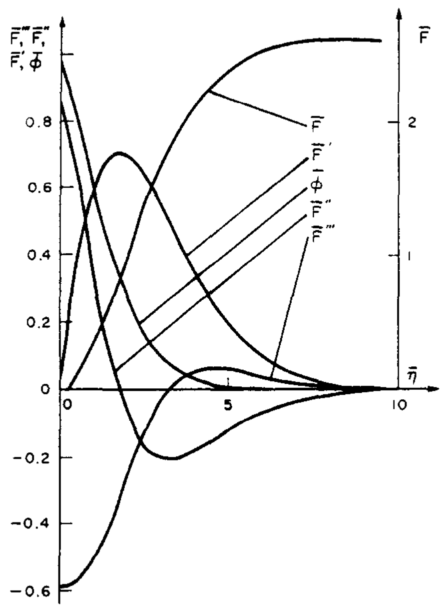


FIG. 1. Analog solution for $Sc = 2.5$ and $y_{A_0} = 0.0005$.

computer — which allows immediate expansion of the $\bar{\eta}$ interval — revealed that $\bar{\eta} = 10$ was required in this problem. Figure 1 displays the solution for the principal case of $Sc = 2.5$ and $y_{A_0} = 0.0005$, which provides the comparison with experimental results. It is clear from this that the asymptotic approach of $\bar{\Phi}$, $\bar{F}'$ and $\bar{F}''$ to 0 is met at $\bar{\eta} = 10$.

On the analog computer, when solutions were run out to a value of only 4 for $\bar{\eta}$ the results of [2] were essentially duplicated. At this inadequately low value for $\bar{\eta}$, the three functions concerned are not asymptotically approaching 0, but have been forced to spot values of exactly zero — see Figs. 13 and 14 of [2].

3. REVISED COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

In Fig. 15 of [2], the authors compared their numerically-produced results with experimental results of four separate investigations, and concluded that the results were in close agreement. With the modified value for $\bar{\Phi}'(0)$ of -0.410, the correlation equation (56) yields now

$$Sh = -\frac{5}{3}(Gr_m\, Sc)^{1/5}\, \bar{\Phi}'(0) = 0.6833\ (Sc\, Gr_m)^{1/5}.$$

Figure 2, a revision of Fig. 15 of [2], shows this corrected correlation line. The agreement between theoretical and experimental results is obviously enhanced by the correction of the error in the solution.

It should also be noted that the scale for Sherwood number, as published at Fig. 15 of [2], is wrong by a factor of 10.

4. ANALOG PROGRAM AND CIRCUIT

Using the method of normalised variables for magnitude scaling, expected maximum magnitudes were chosen to be

$$\bar{F}'''_m = 0.6,\ \bar{F}''_m = 1,\ \bar{F}'_m = 0.7,\ \bar{F}_m = 3,$$
$$\bar{\Phi}'_m = 0.5,\ \bar{\Phi}_m = 1,\ \bar{\eta}_m = 10.$$

Normalisation to those values produces the scaled equations

$$-\left(\frac{\bar{F}'''}{0.6}\right) = -\int_0^t \left[-3.333\left(\frac{\bar{\Phi}'}{0.5}\right)\left(\frac{\bar{\eta}}{10}\right) - 0.72\left(\frac{\bar{F}'''}{0.6}\right)\left(\frac{\bar{F}}{3}\right) - 0.0933\left(\frac{\bar{F}''}{1}\right)\left(\frac{\bar{F}'}{0.7}\right)\right]\mathrm{d}t - \left(\frac{\bar{F}'''}{0.6}\right)_{t=0},$$

$$\left(\frac{\bar{F}''}{1}\right) = -0.6\int_0^t -\left(\frac{\bar{F}'''}{0.6}\right)\mathrm{d}t + \left(\frac{\bar{F}''}{1}\right)_{t=0},$$

$$-\left(\frac{\bar{F}'}{0.7}\right) = -1.429\int_0^t \left(\frac{\bar{F}''}{1}\right)\mathrm{d}t - \cancelto{0}{\left(\frac{\bar{F}'}{0.7}\right)_{t=0}}$$

$$\left(\frac{\bar{F}}{3}\right) = -0.2333\int_0^t -\left(\frac{\bar{F}''}{1}\right)\mathrm{d}t + 0.0001390\left(\frac{\bar{\Phi}'}{0.5}\right)_{t=0},$$

$$-\left(\frac{\bar{\Phi}'}{0.5}\right) = -1.8\int_0^t -\left(\frac{\bar{F}}{3}\right)\left(\frac{\bar{\Phi}'}{0.5}\right)\mathrm{d}t - \left(\frac{\bar{\Phi}'}{0.5}\right)_{t=0},$$

$$\left(\frac{\bar{\Phi}}{1}\right) = -0.5\int_0^t -\left(\frac{\bar{\Phi}'}{0.5}\right)\mathrm{d}t + \cancelto{1}{\left(\frac{\bar{\Phi}}{1}\right)_{t=0}}$$

$$\left(\frac{\bar{\eta}}{10}\right) = -\int_0^t -0.1\,\mathrm{d}t.$$

for which the analog circuit is given in Fig. 3.

Because of the nonlinearity and the fact that three "initial" conditions are missing, it is best to conduct an exploratory run with all the variables deliberately overscaled by a considerable factor. With this extra room for manoeuvre, one can attain the solution more readily. Having discovered preliminary values for the three missing initial conditions, one rescales in accordance with the maxima just discovered and runs again in order to increase the accuracy. This is the procedure followed here, and the equations above are the final, rescaled, version.

Since the terminal value for $\bar{F}'$ was most affected by the initial value for $\bar{F}'''$, an automatic iteration circuit was incorporated using a memory-pair and integrator No. 8, cycled between HOLD and OPERATE. This circuit adjusted $\bar{F}'''(0)$ continuously to maintain $\bar{F}'(10) = 0$. The subsequent adjustment of potentiometers No. 1 and No. 2 manually, in order to meet the other two downstream conditions, was readily accomplished without the further elaboration of

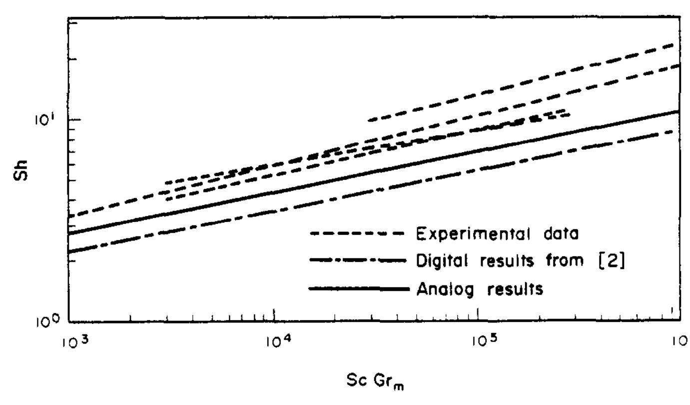
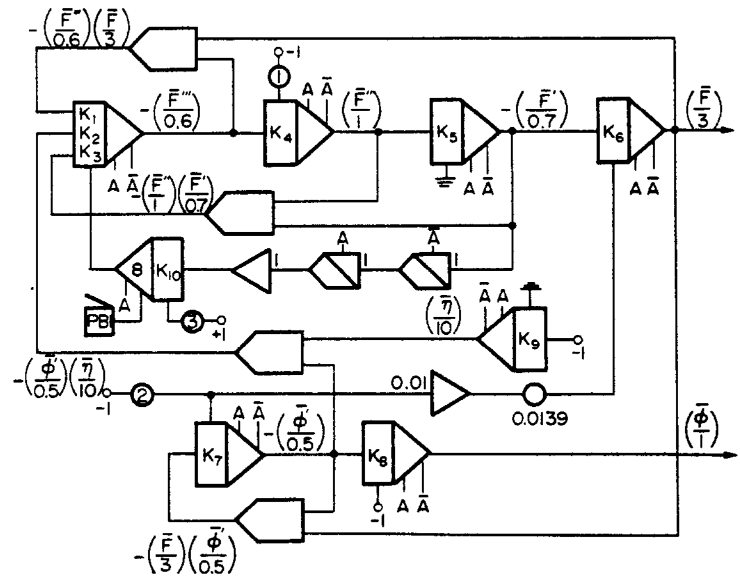


FIG. 2. Revised comparison of theoretical and experimental results.



	Ā	A	K1	K2	K3	K4	K5	K6	K7	K8	K9	K10
Slow	10s	5s	0.72	3.333	0.0933	0.6	1.429	0.2333	1.8	0.5	0.1	0.03
Fast	2ms	1ms	3600	16667	467	3000	7150	1167	9000	2500	500	150

FIG. 3. Analog circuit for $Sc = 2.5$ and $y_{A_0} = 0.0005$.

additional iteration circuits.

Figure 1 was recorded in SLOW mode, the run time of 10 s being suitable to the recorder. By going to FAST mode, the complete solution was accomplished 333 times/s, yielding a static set of curves on an oscilloscope.

A convincing case has been made for the use of analog computers in problems of this nature. The preparatory time in scaling, programming and implementing the solutions is certainly no more than is involved in a digital treatment. The accuracy achieved (around 0.25 %) is perfectly acceptable for any practical purpose, while the run time is dramatically shorter. Even with a small-scale, medium-speed machine like the EAI 380, the solutions can be repetitively displayed hundreds of times/s. By alteration of the appropriate potentiometer(s), the effects of varying a parameter such as Sc or y_{A_0} can be immediately observed.

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2. J. Bandrowski and W. Rybski, Free convection mass transfer from horizontal plates, *Int. J. Heat Mass Transfer* **19**, 827 (1976).